RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2023

FIRST YEAR [BATCH 2022-25]

COMPUTER SCIENCE [HONOURS]

Date : 26/05/2023

Time : 11.00 am – 1.00 pm

Paper : CC4

Full Marks : 50

Answer **any five** questions: [5×10]

1. a) Suppose the set S_n contains all the bit strings of length i, where i = 0, 1, 2,..., n. Again let $P_i = S_i - S_{i-1}$, i = 1, 2, 3, ..., n. Then show that the sets P_i , i = 1, 2, ... , n form the partition of the set S_n .

   b) Let A, B, C be subsets of a universal set S , then prove that

   $(A - B) \times C = (A \times C) - (B \times C)$

   c) What do you mean by equivalence class? Explain with an example.

   d) If the set of real numbers $A_i = [1,i]$ then evaluate the values of

   i) $A_i \cup A_j$, i<j ii) $A_i - A_j$, i > j [3+3+2.5+1.5]

2. a) Using appropriate procedure , find the matrix represents the transitive closure of the relation R= {(a,b), (b,c), (c,a)} defined on the set S={a, b, c}.

   b) Let S be the set of all positive divisors of 45. Define a relation "≥" on S by " x ≥ y if and only if x is divisible by y " for x, y ϵ S. Prove that (S, ≥) is a poset. If possible then draw the corresponding Hasse diagram.

   c) Find a closed form for the generating function for the following sequence

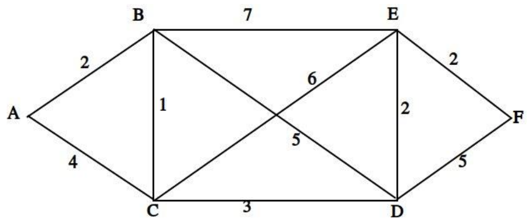
   0, 1, -1, 1, -1, . . . [3+5+2]

3. a) State and prove the general principle of inclusion and exclusion.

   b) Show that in any set of eleven integers, there are two whose difference is divisible by 10.

   c) Show that the relation P on the set R of real numbers defined as $P = \langle (a,b) : a \le b^2 \rangle$ is neither reflexive, nor symmetric, nor transitive. [4+3+3]

4. a) Using appropriate method, find the length of the shortest path between vertices A and F for the following graph. Show the necessary steps.



   b) Define isomorphic graphs with an example.

   c) In an incidence matrix, if all the entries in a row are zero what does it mean ? [6+3+1]

5. a) Define Ring.

   b) Prove that a simple graph with n vertices and k components can not have more than $\frac{(n-k)(n-k+1)}{2}$ edges.

c) Out of 500 car owners investigated, 400 owned Maruti, 200 owned Hyundai, 50 owned both cars. Is this data correct?

d) Define Euler graph. Draw a graph which is Eulerian but not Hamiltonian. [2+4+2+2]

6. a) Using characteristic roots method, solve the following linear non homogeneous recurrence relation.

$a_{n+2} - 4a_{n+1} + 4a_n = 2^n$, with initial condition $a_0 = 1$ and $a_1 = -1$.

b) If X is a binomial variate with $p = \frac{1}{5}$, for the experiment of 50 trials, find the standard deviation.

c) If X is a Poisson variate much that $p(x=2) = p(x=3)$, then find the mean, variance and $p(x=0)$. [5+2+(2+1)]

7. a) Let * be a binary operation on Q defined by $a * b = 2a + b$, $a, b \in Q$. Show that $(Q, *)$ is a quasigroup but not semigroup.

b) If a,b are two positive fixed integers and $S = \langle ax + by : x, y \in z \rangle$,show that S is a subgroup of the group $(Z, +)$.

c) If $f : R \to R$ be a function such that $f(x) = 3x + 5$, prove that f is one-one, onto. [4+3+3]

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